MODIFIED EIGENFILTER APPROACH FOR DESIGNING DIGITAL FULL-BAND DIFFERENTIATOR OF ARBITRARY ORDER

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Key words: Digital finite impulse response (FIR), full-band differentiator, Differentiation order, Differentiator length, Magnitude response, Group delay response.

The method proposed in this paper can be successfully used for the design of the digital FIR full-band differentiator of arbitrary order. It involves the approximation of the FIR digital differentiator frequency response directly in the complex domain, and represents properly modified eigenfilter method. Introduction of the condition that specification parameter of the group delay level τ has a non integer value allows this method to use the same design procedure and formulas for differentiators with both even and odd length, with very small passband magnitude approximation error and approximately constant passband group delay level. These characteristics are enabled due to the fact that differentiators, designed by the presented method, possess neither the (anti)symmetric feature of impulse response coefficients, nor the strictly linear phase.

1. INTRODUCTION

FIR digital differentiator (DD in the subsequent text) is a digital filter whose output signal are samples of the derivative of a band limited continuous time signal. DDs have a broad application in various practical signal processing systems, particularly in instrumentation, radars and motion control systems, where the calculation of the instantaneous rate of sampled signal change is needed. In a general case, DD can be designed by using available numerical differentiation formulas, such as: Gregory-Newton forward and backward difference formulas or Bessel, Everett and Stirling central difference formulas. Several very good approaches for designing the linear-phase DD, which can be reduced on the eigen-filter method, are presented in [1–6]. It is well known that for the design of the FIR filter having lower time delay than the linear phase FIR filter, and approximately

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constant passband group delay level, it is needed to solve the complex approximation problem. Very good techniques for this purpose, are presented in [7–12]. The common characteristic for all of these methods is that their design procedures and equations differ for cases of even and odd filter length, as well as FIR filters designed by these methods principally have the passband group delay level corresponding to the group delay level of the linear phase FIR filter.

One of relevant features of the method proposed in this paper is that it uses the same procedure and formulas for the design of DDs with both even and odd length, without any mutual difference for both length cases. This advantage is enabled by introducing the condition that the specification parameter of the group delay level τ has a non integer value (which is a strict demand in some applications). In addition, proposed method enables to design even the first order FIR full-band DD with odd length N, as well as the second order FIR full-band DD with even length N, which is not possible dealing with FIR full-band DD with the strictly linear phase. The passband group delay level of designed full-band DDs is approximately constant and distinct from that of the corresponding linear phase full-band DDs, while their magnitude response passband approximation error is extremely low.

2. **THEORETICAL BACKGROUND**

The frequency response of an ideal full-band differentiator ($\omega \in [0;\pi)$) with linear phase, of order k, in a general case is given by:

$$F_k(\omega) = (\mathrm{j}\omega)^k \exp(-\mathrm{j}\omega\tau) = M_k(\omega)\exp[\mathrm{j}P_k(\omega)] = F_{Rk}(\omega) + \mathrm{j}F_{Ik}(\omega) = \\ = \begin{cases} \mathrm{j}(-1)^{(k-1)/2}\,\omega^k \exp(-\mathrm{j}\omega\tau) & ; \quad k - \text{odd} \\ (-1)^{k/2}\,\omega^k \exp(-\mathrm{j}\omega\tau) & ; \quad k - \text{even}, \end{cases} \qquad (1)$$

where $M_k(\omega)$ and $P_k(\omega)$ are the magnitude and the phase response of the ideal full-band differentiator, respectively, whereby FIR structure with length N and a linear phase has the passband group delay level given by: $\tau = (N-1)/2$. (To avoid a possible confusion, it is needed here to clarify: in this paper the term "order" is used to indicate the order of the differentiation, while the term "length" is used for specifying the length N of the FIR structure.) In practical use, the need for the FIR full-band digital differentiator of order greater than second, is not very common.

The designed FIR full-band DD with length N and a real impulse response $a(n)$, $n = 0,1,..., N$-1, regardless of its order k, has the frequency response given by:

$$H(\omega) = \sum_{n=0}^{N-1} a(n)\exp(-\mathrm{j}\omega n) = H_R(\omega) + \mathrm{j}H_I(\omega) = \\ = \sum_{n=0}^{N-1} a(n)\cos(n\omega) - \mathrm{j}\sum_{n=0}^{N-1} a(n)\sin(n\omega). \qquad (2)$$

By introducing following vectors:

$$\begin{aligned} \boldsymbol{a} &= [a(0)\ a(1) \ldots a(N-1)]^{\mathrm{T}} \\ \boldsymbol{c}(\omega) &= [1\ \cos(\omega) \ldots \cos[(N-1)\omega]]^{\mathrm{T}} \\ \boldsymbol{s}(\omega) &= [0\ \sin(\omega) \ldots \sin[(N-1)\omega]]^{\mathrm{T}}, \end{aligned} \tag{3}$$

where the superscript T denotes the vector transpose operation, and then applying and substituting expressions (3) in equation (2), it becomes:

$$H(\omega) = \boldsymbol{a}^{\mathrm{T}}\boldsymbol{c}(\omega) - \mathrm{j}\boldsymbol{a}^{\mathrm{T}}\boldsymbol{s}(\omega) = H_R(\omega) + \mathrm{j}H_I(\omega), \tag{4}$$

where, obviously is:

$$\begin{aligned} H_R(\omega) &= \boldsymbol{a}^{\mathrm{T}}\boldsymbol{c}(\omega) \\ H_I(\omega) &= -\boldsymbol{a}^{\mathrm{T}}\boldsymbol{s}(\omega). \end{aligned} \tag{5}$$

Expression (4) is used for the simultaneous approximation both the desired magnitude M_k (ω), and the phase P_k (ω) response of the ideal full-band DD, from (1). In fact, in the defined, full frequency band, the real $H_R(\omega)$ and the imaginary H_I (ω) part of the designed frequency response from (5), are designed to respectively approximate the real $F_{Rk}(\omega)$ and imaginary $F_{Ik}(\omega)$ part of the ideal frequency response, from (1). This approximation is performed by minimization of the quadratic measure error, which is defined on a proper manner [13, 14] as:

$$\begin{aligned} E(\alpha_R;\alpha_I) = {} & \alpha_R \int_0^{\pi} [F_{Rk}(\omega)H_R(\omega_0) - H_R(\omega)F_{Rk}(\omega_0)]^2 \,\mathrm{d}\omega + \\ & + \alpha_I \int_0^{\pi} [F_{Ik}(\omega)H_I(\omega_0) - H_I(\omega)F_{Ik}(\omega_0)]^2 \,\mathrm{d}\omega, \end{aligned} \tag{6}$$

which introduces the modification of the classical eigenfilter method [15] by introduction of weighting coefficients for the separate, but not mutually independent approximation of the real and imaginary part of the frequency response of designed FIR DD. Further modifications and features are presented as follows. Equation (6) can be written as:

$$E(\alpha_R;\alpha_I) = \alpha_R E_R + \alpha_I E_I, \tag{7}$$

where $E(\alpha_R;\alpha_I)$ is the total approximation error (of the total frequency response), ω_0 is the passband referential frequency, α_R and α_I are weighting coefficients of the frequency response real and imaginary part approximation, respectively. E_R and E_I are approximation errors of the frequency response real and imaginary part, respectively, while τ is the desired passband group delay level, from expression (1). It is obvious, from equation (7), that contributions of the frequency response real and imaginary parts approximation errors (E_R and E_I) to the total approximation error $E(\alpha_R;\alpha_I)$, can be adjusted by the pertinent choice of weighting coefficients (α_R, α_I) numerical values (thereby ensuring that: $0 < \alpha_R < 1$; $0 < \alpha_I < 1$ and $\alpha_R + \alpha_I = 1$). This is the main reason of their introducing in expression (6), i.e.(7). Substituting expressions for $F_{Rk}(\omega)$ and $F_{Ik}(\omega)$ from (1), and expressions for $H_R(\omega)$ and $H_I(\omega)$ from (5), into equation (6), one can obtain:

$$E(\alpha_R;\alpha_I) = \boldsymbol{a}^{\mathrm{T}} \left[\alpha_R \boldsymbol{Q_R} + \alpha_I \boldsymbol{Q_I} \right] \boldsymbol{a} . \tag{8}$$

From (7) and (8), it is obvious:

$$\begin{aligned} E_R &= \boldsymbol{a}^{\mathrm{T}} \boldsymbol{Q_R} \boldsymbol{a} \\ E_I &= \boldsymbol{a}^{\mathrm{T}} \boldsymbol{Q_I} \boldsymbol{a}, \end{aligned}, \tag{9}$$

where $\boldsymbol{Q}_R$ and $\boldsymbol{Q}_I$ are $N\times N$ quadratic, real and symmetric matrices of the frequency response real and imaginary part approximation, respectively, while:

$$\boldsymbol{Q} = \alpha_R \boldsymbol{Q_R} + \alpha_I \boldsymbol{Q_I} , \tag{10}$$

represents NxN quadratic, real and symmetric matrix, whose elements and eigen-system are necessary to determine. Thus, expression (8) becomes:

$$E(\alpha_R;\alpha_I) = \boldsymbol{a}^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{a}, \tag{11}$$

which finally represents the classical formulation of the eigenfilter problem in the least-squares sense [15].

Obtained expressions for matrix $\boldsymbol{Q}$ elements, have shown that their values depended on values of specification parameters: α_R, α_I, τ, ω_0 and N. After computing numerical values of matrix $\boldsymbol{Q}$ elements, next step is computing its eigen-values and eigen-vectors. The eigen-vector $\boldsymbol{a}$ of the matrix $\boldsymbol{Q}$, from expression (11), corresponding to its smallest eigen-value, is the vector which minimizes the error (11), i.e. (6), and, accordingly to this, it represents the desired impulse response coefficient vector of the designed full-band DD, from (2).

3. **DISCUSSION AND RESULTS**

Extensive and detailed examinations in connection with the influence of design specification parameters (α_R, α_I, τ, ω_0, N) and their numerical values on magnitude and group delay response approximation errors were performed through designing a large number of numerical examples of first and second order full-band DDs. These examinations have shown that the choice of numerical values, for all specification parameters, could not be done mutually completely independently. In fact, by the proper analysis of frequency response real and imaginary parts approximation errors, values of specification parameters giving a small variation of the group delay response, simultaneously with a small passband magnitude response error, can be obtained. In accordance with this, the choice of these two characteristics assumes a mild compromise between their qualities (which is well known fact). Performed examinations have shown also that, in general, the real part approximation error E_R has a greater contribution to the total approximation error (than imaginary part approximation error E_I). Due to this and according to equation (7), the numerical value of its corresponding weighting coefficient α_R has to be smaller than the value of the weighting coefficient α_I.

The following relevant feature, characterizing and distinguishing the presented method, deserves to be emphasized: this method uses the same procedure and formulas for the design of DDs with both even (Examples 1 and 3) and odd (Examples 2 and 4) length N (see Appendix), without any mutual differences for both length cases. By introducing the condition that parameter of the group delay level τ has a non integer value (which is a strict demand in some applications), even first order FIR full-band DDs with odd length N (Example 2), as well as second order FIR full-band DDs with even length N (Example 3) can be designed, which is not possible dealing with the FIR full-band DD with the strictly linear phase. These facts indice on the proposed method generality. In addition, full-band DDs (Figs. 1.1 and 3.1) designed by the presented method, have approximately constant pass-band group delay level, which can be varied in a relatively wide range, (Figs. 1.3; 2.2; 3.3 and 4.2), as well the extremely low magnitude response error which is below 0,3% in the most of the passband, and does not exceed the value of 1% on passband edges (Figs. 1.2; 1.4; 2.1; 2.3; 3.2 and 4.1). The magnitude response error for the first order FIR DD with strictly linear phase and even length N = 32, designed by the Mc Clellan-Parks algorithm (Chebyshev approximation), has minimax relative error of magnitude response approximation, which ranges from –0,6% to +0.6% between passband edges. The method is simple and fast, and thus, efficient. Designed full-band DDs do not possess the (anti)symmetric feature of their impulse response coefficients, nor the strictly linear phase. All these features are achieved due to introduced modifications of the classical eigenfilter method.

4. CONCLUSIONS

The method proposed in this paper can be successfully used for the design of the full-band DD of (theoretically) arbitrary order. By introducing the condition that specification parameter of the group delay level τ has a non integer value (which is a strict demand in some applications), it is possible to design even the first order FIR full-band DD with odd length N, as well as the second order FIR full-band DD with even length N, which is not possible dealing with the digital FIR full-band DD with the strictly linear phase. In addition, this method uses the same procedure and formulas for the design of differentiators with both even and odd length (without any mutual difference),which indices on the proposed method generality. Mentioned characteristics are enabled due to the fact that full-band DD, designed by the presented method, do not possess the (anti)symmetric feature of their impulse response coefficients, nor the strictly linear phase. They have approximately constant pass-band group delay level, which can be varied in a relatively wide range. They also have extremely low magnitude response error, which is below 0,3% in the most of the passband, and does not exceed the value of 1% on passband edges.

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APPENDIX

Example 1. Design of the first order FIR full-band DD with the even length N and following specification parameters values: $N = 32$; $\tau = 9.5$; $\omega_0 = 0.5\pi$; $\alpha_I = =99\alpha_R$. (Figs. 1.1, 1.2, 1.3 and 1.4).

Example 2. Design of the first order FIR full-band DD with the odd length N and following specification parameters values: $N = 31$; $\tau = 11.5$; $\omega_0 = 0.5\pi$; $\alpha_I = =99\alpha_R$. (Figs. 1.1, 2.1, 2.2 and 2.3).

Example 3. Design of the second order FIR full-band DD with the even length N and following specification parameters values: $N = 32$; $\tau = 12.5$; $\omega_0 = =0.5\pi$; $\alpha_I = 99\alpha_R$. (Figs. 3.1, 3.2, 3.3).

Example 4. Design of the second order FIR full-band DD with the odd length N and following specification parameters values: $N = 31$; $\tau = 11.5$; $\omega_0 = 0.5\pi$; $\alpha_I = =99\alpha_R$. (Figs. 3.1, 4.1, 4.2).

Note. Magnitude responses of first order FIR full-band DDs from Examples 1 and 2, have the same graphic appearance. Due to this (and the limited manuscript volume), only one magnitude response graphic, presented in the Fig. 1.1, is given and holds for both Examples 1 and 2.

The same feature holds for the magnitude responses of second order FIR full-band DDs from Examples 3 and 4: accordingly, only one magnitude response graphic, presented in the Fig. 3.1, is given and holds for both Examples 3 and 4.

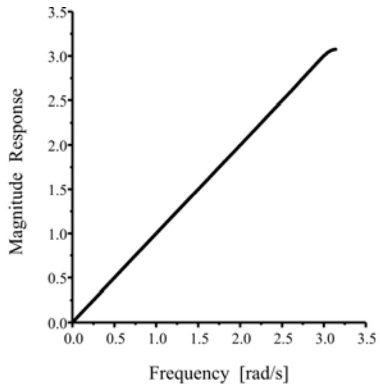

Fig. 1.1 – Magnitude response of the first order FIR full-band DD from Examples 1 and 2.

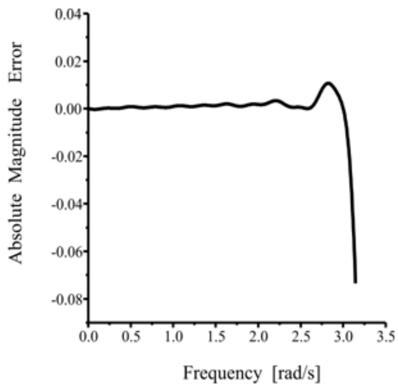

Fig. 1.2 – Absolute error of the magnitude response of the DD from Example 1.

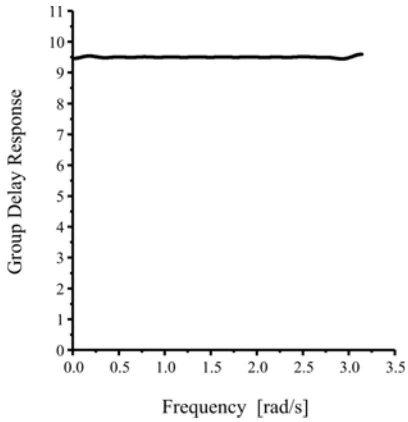

Fig. 1.3 – Group delay response of the DD from Example 1.

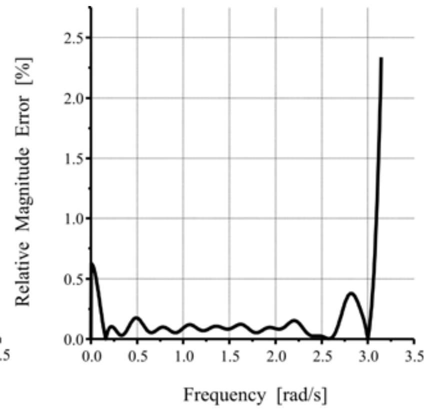

Fig. 1.4 – Relative error of the magnitude response of the DD from Example 1.

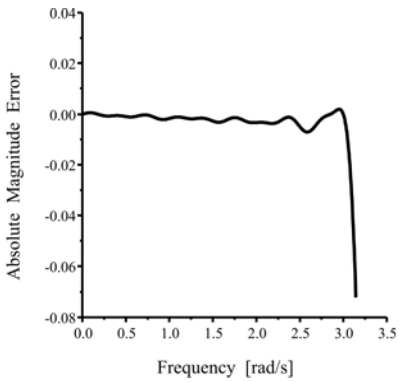

Fig. 2.1 – Absolute error of the magnitude response of the DD from Example 2.

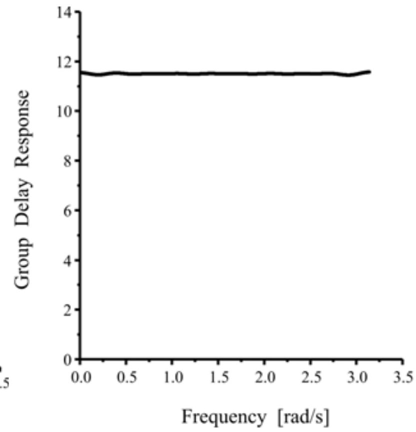

Fig. 2.2 – Group delay response of the DD from Example 2.

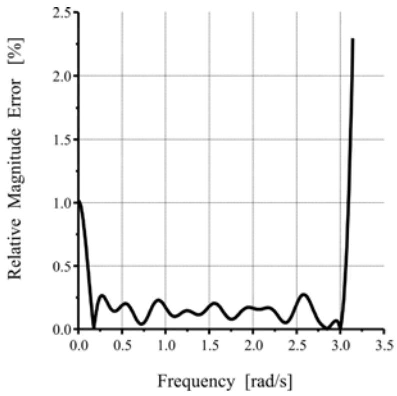

Fig. 2.3 – Relative error of the magnitude response of the DD from Example 2.

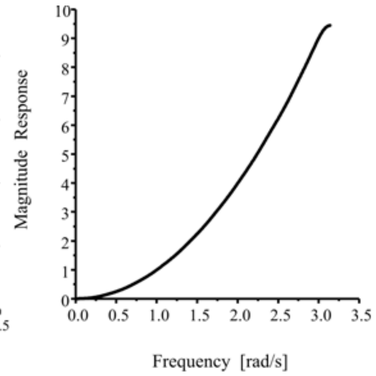

Fig. 3.1 – Magnitude response of the second order FIR full-band DD from Examples 3 and 4.

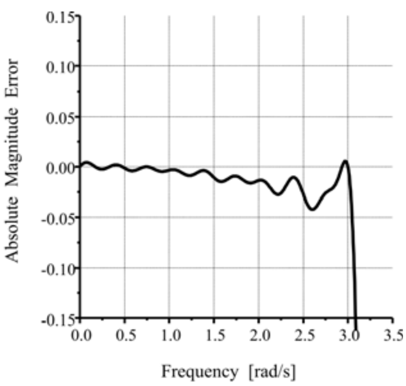

Fig. 3.2 – Absolute error of the magnitude response of the DD from Example 3.

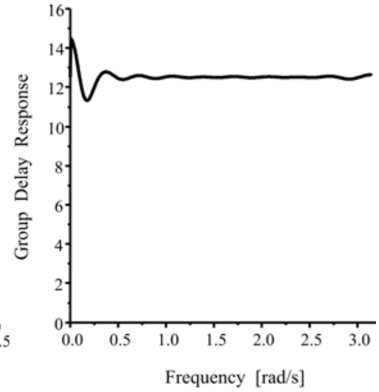

Fig. 3.3 – Group delay response of the DD from Example 3.

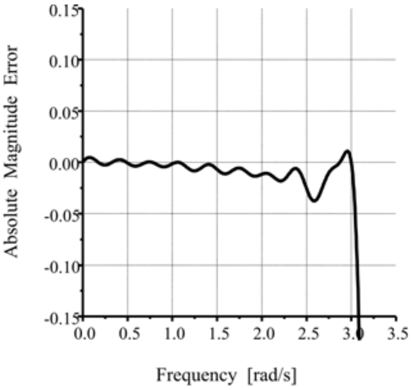

Fig. 4.1 – Absolute error of the magnitude response of the DD from Example 4.

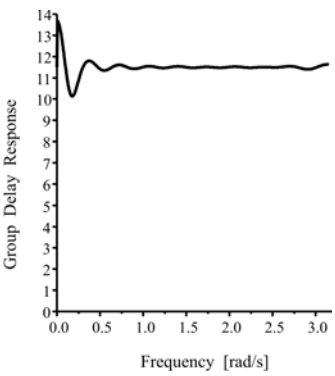

Fig. 4.2 – Group delay response of the DD from Example 4.

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